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Research Article

Influence of Non -thermal Electron Parameter on Heavier Masses of Negative Ion Plasma

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Abstract

In presence of non-thermal electrons, theoretical investigations on ion-acoustic solitary waves are made in a collisionless unmagnetized warm plasma consisting of positive and negative ions by the well known pseudopotential technique. The influence of non-thermal electron parameter(β) on heavier masses (Q) of negative ion plasma for Sagdeev potential function$[\psi(\phi)]$, first $(\phi_1)$and second (ϕ_2)order solitary wave solutions are mainly analysed and discussed here properly.

Keywords: Pseudopotential method, non-thermal electron, solitary waves, drift motion, heavier masses of negative ion plasma.

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1.   INTRODUCTION

Ion-acoustic solitary waves have been investigated theoretically by a large number of scientists[1-4] in presence of isothermal or non-isothermal electrons with warm positive as well as warm negative ion drifts in Maxwellian distribution, but the characteristics of space plasmas are generally controlled by collective behaviour of wave-particle interaction with the exhibition of distinct non-Maxwellian high energy tail distribution. Elwakil et al[5] considered non-thermal(β) Cairn's type electron with positive and negative ion plasma from non-Maxwellian distribution. In upper ionosphere, solitary waves with electrostatic structures involving density depletions have been observed by Freja satellite[6]. On the other hand, Lonngren[7],Ikezi[8] and Ikezi et al[9] investigated the same ion-acoustic solitary waves experimentally with a dissimilarity of amplitude and width from theoretical values. Introducing different plasma parameters, a group of scientists[10-13] have done theoretically a lot of noble works on the propagation of ion-acoustic solitary waves for isothermal plasma($\beta = 0$) in presence of positive and negative ions. In an unbounded plasma with warm non-thermal electrons(β), both compressive as well as rarefactive solitary waves are found in presence of positive ions and negative ions because non-thermal plasma plays an important role for the formation of both solitary waves. Cairns et al[14] showed in their paper that the presence of non-thermal electrons may change the nature of ion-acoustic solitary waves and allow the existence of their structures very like those observed. They have shown that the solitons with both positive and negative density perturbations can exist in presence of non-thermal electron population. Bhattacharya et al[15] investigated the effects of non-thermal electrons (β)and negative ions on ion-acoustic solitary waves in a bounded plasma and found some important results based on compressive and rarefactive solitary waves. Again Paul et al[16] studied analytically the non-thermal plasma with positive ion only. The present author also investigated the same non-thermal plasma with positive and negative ion drifts highlighting the higher order solitary wave solutions, stability of these solutions and comparison of first and second order amplitudes and widths in presence[17] and in absence[18] of positron. Regarding this non-thermal electron plasma, earlier authors did not take the heavier masses of negative ion plasma and also did not show the effect of non-thermal electron (β)on heavier negative ion masses(Q). The present author thus tried to take the heavier negative ion plasma(Q) and studied the influence of non-thermal electron parameter(β) on heavier masses of negative ion plasma.

The working plan of the present paper is arranged in the following way: In sec.2 we investigate the exact form of the Sagdeev potential function $\psi(\varphi)$ in warm and cold ion plasma with an analytical calculation of the max. Value of the electrostatic potential (ϕ_m) i.e. amplitude where $\psi(\phi_m) = 0$, first (ϕ_1) and second (ϕ_2) order solitary wave solutions with the conditions for the existence of their stable solutions in both warm and cold ion plasma. The critical value of the heavier masses of negative ion plasma (Q_c) is calculated in this section for both warm and cold ion plasma. The entire problem is discussed very carefully in sec.3. In this section graphical representation of the sagdeev potential function $\psi(\varphi)$ against electrostatic potential (φ) is discussed with the variation of different plasma parameters like non-thermal parameter(β), ratio of negative to positive ion masses plasma (Q) and negative ion concentration (n_{jo}). The graphical nature of first $(\phi_1)$and second order(ϕ_2) solitary wave solutions are analysed here with great care in this section on the basis of the variation of heavier masses of negative ion plasma(Q). Concluding remarks are given in sec.4.

2. FORMULATION

We consider a collisionless unmagnetized plasma consisting of warm non-thermal electrons, positive and negative ions with drifts. The governing normalised basic set of equations along x-axis for such types of unbounded plasmas are

Equation of continuity: $\dfrac{\partial n_\alpha}{\partial t} + \dfrac{\partial}{\partial x}\left(n_\alpha u_\alpha\right) = 0$ \hfill (1)

Equation of motion: $\dfrac{\partial u_\alpha}{\partial t} + u_\alpha \dfrac{\partial u_\alpha}{\partial x} + \dfrac{\sigma_\alpha}{Q_\alpha n_\alpha}\dfrac{\partial p_\alpha}{\partial x} = -\dfrac{Z_\alpha}{Q_\alpha}\dfrac{\partial \phi}{\partial x}$ \hfill (2)

Pressure equation: $\dfrac{\partial p_\alpha}{\partial t} + u_\alpha \dfrac{\partial p_\alpha}{\partial x} + 3 p_\alpha \dfrac{\partial u_\alpha}{\partial x} = 0$ \hfill (3)

Poisson's equation: $\dfrac{\partial^2 \phi}{\partial x^2} = n_e - \sum_\alpha Z_\alpha n_\alpha$ \hfill (4)

where $n_\alpha, u_\alpha, \sigma_\alpha, Q_\alpha, p_\alpha, Z_\alpha, n_e, \phi$, t and x are respectively the number density of ions, ion fluid velocity, temperatures of ions, ratio of negative to positive ion masses, pressure of ions, charge of ions, concentration of non-thermal electrons, electrostatic potential, time and distance. The charge neutrality condition is

$$\sum_\alpha n_{\alpha 0} Z_\alpha = n_{e0} \tag{5}$$

$$\text{Or, } n_{i0} = 1 + Z n_{j0} \tag{6}$$

In this case $\sigma_\alpha = \dfrac{T_\alpha}{T_e}$ where T_α is the temperatures of ion and T_e is the temperature of electron,

$Q_\alpha = \dfrac{m_j}{m_i} = 1$ and $Z_\alpha = 1$ for positive ion (i), $Q_\alpha = Q$ and $Z_\alpha = -Z$ for negative ion (j). The

normalized density of non-thermal electron is

$$n_e = (1 - \beta\phi + \beta\phi^2)e^\phi \tag{7}$$

Where $\beta = \dfrac{4\lambda}{1+3\lambda}$ [with $\lambda \geq 0$ and $0 \leq \beta < \dfrac{4}{3}$]

In this case β measures the deviation from the thermalised state and λ determines the presence of fast particles in the model.

In the above equations, we have normalised the densities by the equilibrium value n_0, the velocities by the characteristic value $\sqrt{\dfrac{KT_e}{m_\alpha}}$ where m_α is the mass of ion and K is the Boltzmann constant, the pressure by the ion–equilibrium pressure $p_0 = n_0 T_\alpha$, the potential by $\dfrac{KT_e}{e}$, the time by $\sqrt{\dfrac{m_\alpha}{4\pi e^2 n_o}}$ and the distance by the Debye length $\sqrt{\dfrac{KT_e}{4\pi n_o e^2}}$ so that the equations appear totally in dimensionless form.

For solitary wave solution we assume that the dependent variables depend on a single independent variable η defined by

$$\eta = x - Vt \tag{8}$$

where V is the velocity of the solitary wave .The boundary conditions are

$$n_\alpha \to n_{\alpha o}, u_\alpha \to u_{\alpha o}, p_\alpha \to 1, n_e \to 1, \text{ and } \phi \to 0 \text{ at } |x| \to \infty \tag{9}$$

Following Chattopadhyay[19] and using the above transformation (8) & boundary conditions (9) we get finally from equations (1) to (4)

$$n_\alpha = \frac{1}{2}\sqrt{\frac{Q_\alpha\, n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\sqrt{(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}} - \sqrt{(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}}\right] \tag{10}$$

And

$$\frac{d^2\phi}{d\eta^2} = n_e - \frac{1}{2}\Sigma Z_\alpha\sqrt{\frac{Q_\alpha\, n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\sqrt{(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}} - \right.$$

$$\left. \sqrt{(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}}\right] \tag{11a}$$

By equation (7) we get from equation (11a)

$$\frac{d^2\phi}{d\eta^2} = (1 - \beta\phi + \beta\phi^2)e^\phi - \frac{1}{2}\Sigma Z_\alpha\sqrt{\frac{Q_\alpha\, n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\sqrt{(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}} - \right.$$

$$\left. \sqrt{(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}}\right] \tag{11b}$$

For n_α to be real the following restrictions on ϕ is

$$-\frac{Q}{2Z}\left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 < \phi < \frac{1}{2}\left(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 \tag{12}$$

From equations (1) to (4) after using the boundary conditions (9), the above transformation relation (8) and by the equation (7) we can write finally equation (11b) as

$$\frac{d^2\phi}{d\eta^2} = S_1\phi + S_2\phi^2 + S_3\phi^3 + S_4\phi^4 + \ldots\ldots\ldots\ldots = -\frac{\partial\psi}{\partial\phi} \tag{13}$$

Where $S_1 = (1 - \beta) - \dfrac{n_{io}}{(V - u_{io})^2 - \frac{3\sigma_i}{n_{io}}} - \dfrac{Z^2 n_{jo}}{Q(V - u_{jo})^2 - \frac{3\sigma_j}{n_{jo}}}$

$$S_2 = -\frac{1}{2}\left[\begin{array}{l} 1 + \frac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^{-3} - (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^{-3}\right\} - \\ \frac{Z^3 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{3}{2}}\sqrt{3\sigma_j}}\left\{\left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-3} - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-3}\right\} \end{array}\right]$$

$$S_3 = \frac{1}{6}\left[\begin{array}{l} (1 + 3\beta) + \frac{3n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^{-5} - (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^{-5}\right\} + \\ \frac{3Z^4 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{5}{2}}\sqrt{3\sigma_j}}\left\{\left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-5} - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-5}\right\} \end{array}\right]$$

$$S_4 = \frac{1}{24}\left[\begin{array}{l} (1 + 8\beta) - \frac{15n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^{-7} - (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^{-7}\right\} + \\ \frac{15Z^5 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{7}{2}}\sqrt{3\sigma_j}}\left\{\left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-7} - \left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-7}\right\} \end{array}\right]$$

$$\tag{14}$$

$$\boldsymbol{\psi}(\phi) = (1 + 3\beta) - [(1 + 3\beta) - 3\beta\phi + \beta\phi^2]e^\phi - \frac{n_{io}^{\frac{3}{2}}}{6\sqrt{3\sigma_i}}\left[\{(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi\}^{\frac{3}{2}} - \right.$$

$$\{(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi\}^{\frac{3}{2}} - (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^3 + (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^3\right]$$

$$- \frac{Q^{\frac{3}{2}} n_{jo}^{\frac{3}{2}}}{6\sqrt{3\sigma_j}}\left[\{\left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}\}^{\frac{3}{2}} - \{\left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}\}^{\frac{3}{2}} - \left(V - u_{jo} + \right.\right.$$

$$\left.\left.\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^3 + \left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^3\right]\tag{15a}$$

The above function $\boldsymbol{\psi}(\phi)$ can also be written in the following form:

$$\psi(\phi) = -\frac{1}{2}s_1\phi^2 - \frac{1}{3}s_2\phi^3 - \frac{1}{4}s_3\phi^4 - \frac{1}{5}s_4\phi^5 - \dots\dots\dots\tag{15b}$$

Also the form of the said function $\psi(\phi)$ for cold positive and negative ion plasma is given by

$$\psi(\varphi) = (1 + 3\beta) - [(1 + 3\beta) - 3\beta\varphi + \beta\varphi^2]e^{\varphi} + n_{io}(V - u_{io})^2 \left[1 - \sqrt{1 - \frac{2\varphi}{(V - u_{io})^2}}\right]$$

$$- ZQn_{jo}(V - u_{jo})^2 \left[1 - \sqrt{1 + \frac{2Z\varphi}{Q(V - u_{jo})^2}}\right] \tag{15c}$$

The function $\psi(\varphi)$ in (15) is called the sagdeev potential and it will be reduced to that form of potential which Cairns et al[14] obtained for $n_{jo} = 0$, $n_{io} = 1$ and $u_{io} = 0$. Again when $n_{jo} = 0$(i.e. negative ions are absent) we get that form of $\psi(\varphi)$ that Paul et al[16] obtained. This Sagdeev potential function $\psi(\varphi)$ in (15a) and (15c) is more general form than Ref.[14]. In absence of cold negative ion plasma [i.e.$n_{jo} = 0, \sigma_j = 0$], the above form of Sagdeev potential function $\psi(\varphi)$ and the concentration of ion $n_{\propto}$ are reduced to the form of Ref.[14] provided $n_{io} \rightarrow 1$ and V - u_{io} = M.

Under condition (12) for formation of solitary wave solution $\psi(\varphi) = 0$ at $\varphi = 0$, $\psi(\varphi) = 0$ at $\varphi = \phi_m$ (say) [$\phi_m \neq 0$] where ϕ_m is the max.value of φ and $\psi(\varphi) < 0$ in $0 < |\varphi| < \phi_m$, we now get from (15b) at $\varphi = \phi_m$

$$\frac{1}{2}S_1 + \frac{1}{3}S_2\phi_m + \frac{1}{4}S_3\phi_m^2 + \frac{1}{5}S_4\phi_m^3 = 0$$

For solution of second order solitary wave, taking terms upto ϕ_m^2, we get from the above equation as

$$\frac{1}{2}S_1 + \frac{1}{3}S_2\phi_m + \frac{1}{4}S_3\phi_m^2 = 0$$

The above equation gives

$$\phi_m = \frac{-2S_2 + \sqrt{4S_2^2 - 18S_1S_3}}{3S_3} \quad , \quad \phi_m = \frac{-2S_2 - \sqrt{4S_2^2 - 18S_1S_3}}{3S_3}$$

For compressive solitary waves $\phi_m > 0$ and for rarefactive solitary waves $\phi_m < 0$.

Now from equation (13) taking terms upto ϕ^2 we get

$$\frac{d^2\varphi}{d\eta^2} = S_1\varphi + S_2\varphi^2$$

The first order K-dV soliton solution[20] is

$$\phi_1 = \frac{3S_1}{2S_2} sech^2 \left(\sqrt{\frac{S_1}{4}} . \eta \right)$$ (16)

Again taking terms upto ϕ^3 from (13) we get

$$\frac{d^2\phi}{d\eta^2} = S_1\phi + S_2\phi^2 + S_3\phi^3$$

In this case the higher order M-KdV solitary wave solution[20] is

$$\phi_2 = \frac{6S_1}{2S_2 + \sqrt{4S_2^2 - 18S_1S_3} \left[2Cosh^2 \left(\sqrt{\frac{S_1}{4}} \eta \right) - 1 \right]}$$ (17)

2.1   Condition for solitary wave solution and its consequences

The first order soliton solution (ϕ_1) will be real and finite only when

$S_1 > 0$ and $S_2 \neq 0$ (18)

It is also evident from (18) that the first order solution may be compressive or rarefactive.

Again the second order soliton solution (ϕ_2) will be real and finite only when

$S_1 > 0$ and $4S_2^2 - 18\,S_1S_3 > 0$ (19)

The condition for the existence of solitary wave solution will be obtained from $S_1 > 0$

[i.e. $\frac{\partial^2\psi}{\partial\phi^2} < 0$ at $\phi = 0$] which gives

$$\frac{n_{io}}{(V - u_{io})^2 - \frac{3\sigma_i}{n_{io}}} + \frac{Z^2 n_{jo}}{Q(V - u_{jo})^2 - \frac{3\sigma_j}{n_{jo}}} + \beta < 1$$ (20a)

In absence of cold negative ion ($n_{j0} = 0$, $\sigma_j = 0$) and for $n_{i0} \to 1$, inequality (20a) supports Ref.[16]. Again in another words for absence of non-thermal electron ($\beta = 0$), inequation (20a) supports Ref.[19]. For cold non-thermal plasma [$\sigma_i = 0$, $\sigma_j = 0$], the above condition (20a) reduces to the form

$$\frac{n_{io}}{(V-u_{io})^2} + \frac{Z^2 n_{jo}}{Q(V-u_{jo})^2} + \beta < 1 \tag{20b}$$

The critical value of Q $(= Q_c)$ is obtained from (20a) by the equation

$$Q_c = \frac{1}{(V-u_{jo})^2}\left[Z^2 n_{jo}\left\{(1-\beta) - \frac{n_{io}}{(V-u_{io})^2 - \frac{3\sigma_i}{n_{io}}}\right\}^{-1} + \frac{3\sigma_j}{n_{jo}}\right] \tag{20c}$$

For cold ion plasma i.e. $\sigma_i = 0, \sigma_j = 0$ the expression for Q_c is

$$Q_c = \frac{Z^2 n_{jo}}{(V-u_{jo})^2}\left[\left\{(1-\beta) - \frac{n_{io}}{(V-u_{io})^2}\right\}^{-1}\right] \tag{20d}$$

Again for cold positive and negative ion plasma $[\sigma_i=0, \sigma_j = 0]$, the critical value of β $(= \beta_c)$ is

$$\beta_c = 1 - \frac{n_{io}}{(V-u_{io})^2} - \frac{Z^2 n_{jo}}{Q(V-u_{jo})^2} \tag{20e}$$

Relations (20c) and (20d) are very important because the effect of non-thermal parameter on heavier masses of negative ion plasma is shown by those relations.

Now from (19), $4S_2^2 - 18 S_1 S_3 > 0$ for second order solitary wave solution, we get finally

$$\left[\begin{array}{l} 1 + \frac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V-u_{io}+\sqrt{\frac{3\sigma_i}{n_{io}}})^{-3} - (V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}})^{-3}\right\} - \\[2ex] \frac{Z^3 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{3}{2}}\sqrt{3\sigma_j}}\left\{\left(V-u_{jo}+\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-3} - \left(V-u_{jo}-\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-3}\right\} \end{array}\right]^2 >$$

$$3\left[(1-\beta) - \frac{n_{io}}{(V-u_{io})^2 - \frac{3\sigma_i}{n_{io}}} - \frac{Z^2 n_{jo}}{Q(V-u_{jo})^2 - \frac{3\sigma_j}{n_{jo}}}\right] \times$$

$$\left[\begin{array}{l} (1+3\beta) + \frac{3n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V-u_{io}+\sqrt{\frac{3\sigma_i}{n_{io}}})^{-5} - (V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}})^{-5}\right\} + \\[2ex] \frac{3Z^4 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{5}{2}}\sqrt{3\sigma_j}}\left\{\left(V-u_{jo}+\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-5} - \left(V-u_{jo}-\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^{-5}\right\} \end{array}\right] \tag{21a}$$

This gives the condition for real second order (ϕ_2) soliton solution along with (20a). For cold non-thermal plasma the above condition (21a) reduces to the following form

$$\left[1 - \frac{3n_{io}}{(V - u_{io})^4} + \frac{3Z^3 n_{jo}}{Q^2(V - u_{jo})^4}\right]^2 > 3\left[(1 - \beta) - \frac{n_{io}}{(V - u_{io})^2} - \frac{Z^2 n_{jo}}{Q(V - u_{jo})^2}\right] \times$$

$$\left[(1 + 3\beta) - \frac{15\,n_{io}}{(V - u_{io})^6} - \frac{15\,Z^4 n_{jo}}{Q^3(V - u_{jo})^6}\right] \tag{21b}$$

For cold positive ion non-thermal plasma (i.e. in absence of cold negative ion $n_{jo} = 0$), inequality (21b) reduces to the following inequality

$$\left[1 - \frac{3n_{io}}{(V - u_{io})^4}\right]^2 > 3\left[(1 - \beta) - \frac{n_{io}}{(V - u_{io})^2}\right] \times \left[(1 + 3\beta) - \frac{15\,n_{io}}{(V - u_{io})^6}\right] \tag{21c}$$

The max.value of the electrostatic potential ($\phi = \phi_m$) for second order solitary wave solution so obtained may be either positive (i.e. compressive in nature) or negative (i.e. rarefactive in nature), satisfied all the three cases (21a), (21b) and (21c).

In case of non-drifting positive and negative ion plasma (i.e. $u_{io} = 0$, $u_{jo} = 0$) with   non-thermal electron ($\beta \neq 0$), inequality (21b) reduces to the following form:

$$\left[1 - \frac{3n_{io}}{V^4} + \frac{3Z^3 n_{jo}}{Q^2 V^4}\right]^2 > 3\left[(1 - \beta) - \frac{n_{io}}{V^2} - \frac{Z^2 n_{jo}}{QV^2}\right] \times \left[(1 + 3\beta) - \frac{15\,n_{io}}{V^6} - \frac{15\,Z^4 n_{jo}}{Q^3 V^6}\right] \tag{21d}$$

By equation (6), the inequality (21d) gives the following result for $n_{jo} \rightarrow 1$, $Z \rightarrow 1$
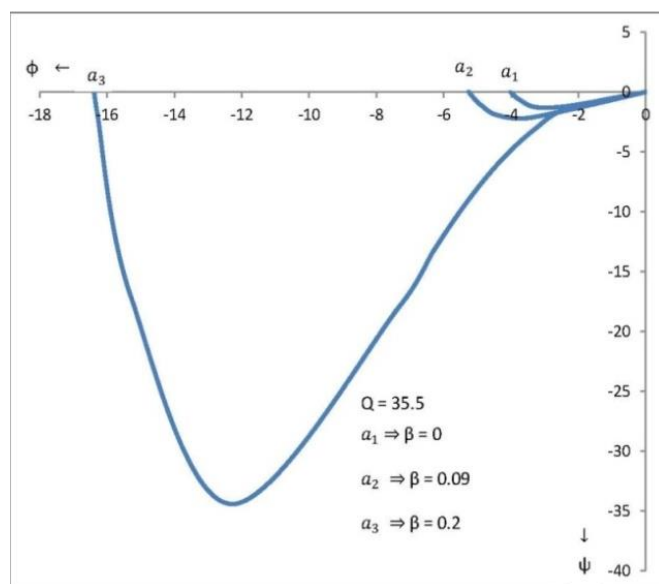
$$\left[1 - \frac{6}{V^4} + \frac{3}{Q^2 V^4}\right]^2 > 3\left[(1 - \beta) - \frac{2}{V^2} - \frac{1}{QV^2}\right] \times \left[(1 + 3\beta) - \frac{30}{V^6} - \frac{15}{Q^3 V^6}\right] \tag{21e}$$

It is an eighth degree inequation in V giving eight real or imaginary values of the phase velocity V depending on the values of Q, β and Z. The influence of Q on V is an interesting case. Inequality (21e) also shows the influence of β on heavier masses of negative ion plasma(Q).

3.   DISCUSSION

It is important to note in this context that heavier masses of negative ion plasma (Q) affect the formation of solitary waves in presence of non-thermal electrons. So we are now discussing about the Sagdeev potential function(ϕ) , first (ϕ_1) and second (ϕ_2) order solitary wave solutions of the heavier masses of negative ion plasma (Q) with non-thermal situation $(\beta \neq 0)$.These are represented by the Figures 1 to 4.

In Figure 1, the profiles of Sagdeev potential function $\psi(\phi)$ verses electrostatic potential (ϕ) with the variation of non-thermal electron parameter (β) are drawn for heavier masses of negative ion plasma (Q). For (H$^+$, Cl$^-$) plasma with mass ratio Q = 35.5, rarefactive solitary waves are obtained in presence $(\beta \neq 0)$ and in absence $(\beta = 0)$ of non-thermal electron plasma. No compressive solitary waves are formed in this case. But it can be found that compressive solitary waves may be present in the system for $\varphi > 0$ under some specific values of the plasma parameters within the specified range. As β increases, the amplitude of the rarefactive solitary waves increases. Moreover it is observed also that the amplitudes of the rarefactive solitary waves with non-thermal electrons $(\beta \neq 0)$ are always greater than that of the amplitudes of the rarefactive solitary waves with isothermal electrons (β=0).The curves a_1, a_2 and a_3 showing the rarefactive solitary waves in Figure 1 represent the respective Sagdeev potential (ψ) function at β = 0, 0.09 and 0.2 with Q = 35.5. It is also seen from those figures that $a_1 < a_2 < a_3$ due to increase of β for a higher mass ratio Q. This is the effect of β on Q. Moreover a comparison between a non-thermal and an isothermal electron plasma is also observed.

Figure 1: ψ verses ϕ for β = 0, 0.09, 0.2

Figure 2 shows the profiles of Sagdeev potential function $\psi(\phi)$ verses electrostatic potential $(\phi)$with the variation of the ratios of heavier negative to positive ion masses (Q) plasma.
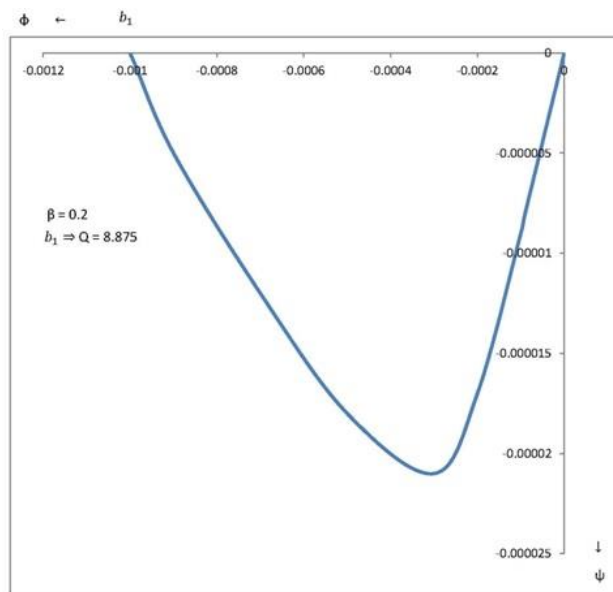
**Figure2(a):** ψ verses ϕ for Q = 8.875
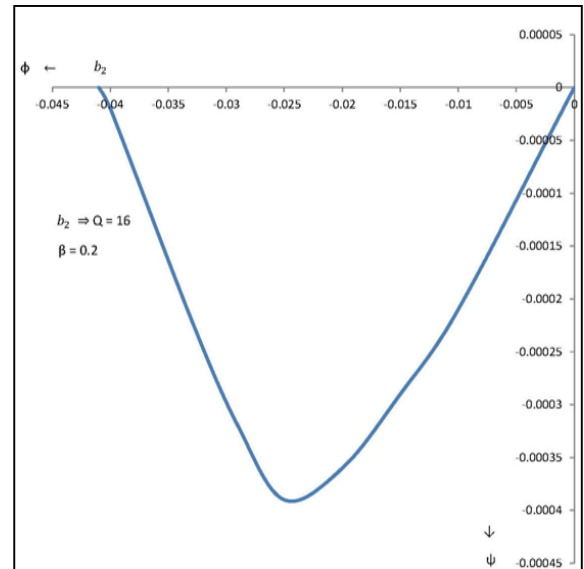
Figure2(b): ψ verses ϕ for Q = 16

In presence of non-thermal electron plasma(for β = 0.2), rarefactive (ϕ < 0) solitary waves b_1, b_2 and b_3 are formed for the relatively heavier masses (Q = 8.875, 16, 35.5) of negative ion plasma which are shown in Figures.2(a), 2(b) and 2(c). The amplitudes of those rarefactive solitary waves are increasing for higher values of Q . Again in absence of non-thermal plasma (β = 0), the amplitudes of the rarefactive solitary waves are increasing similarly for heavier masses of negative ion plasma but at the same time they are higher than those of the values of the amplitudes with non-thermal electron plasma ($\beta \neq 0$).
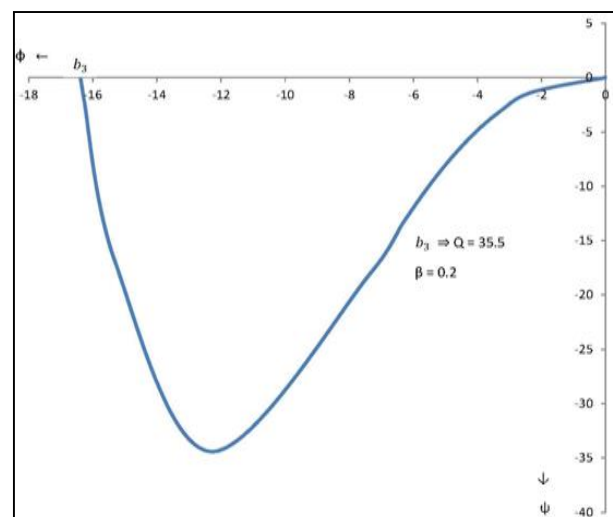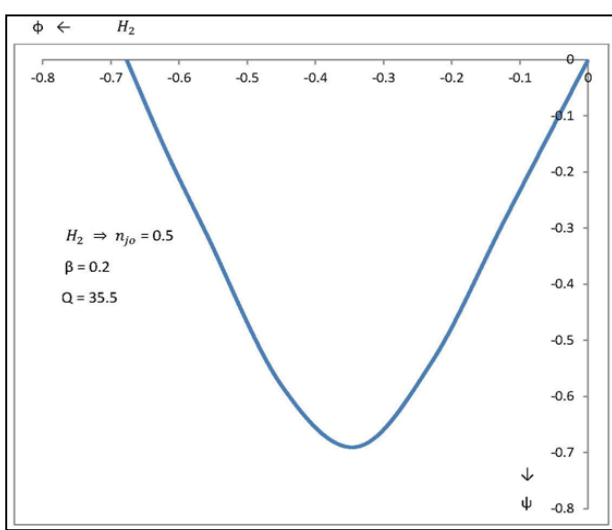

Figure2(c): ψ verses ϕ for Q =35.5

In figure 3, the profiles of Sagdeev potential function $\psi(\phi)$ verses electrostatic potential (ϕ) with the variation of the concentration of heavier negative ion (n_{j0}) plasma are shown.
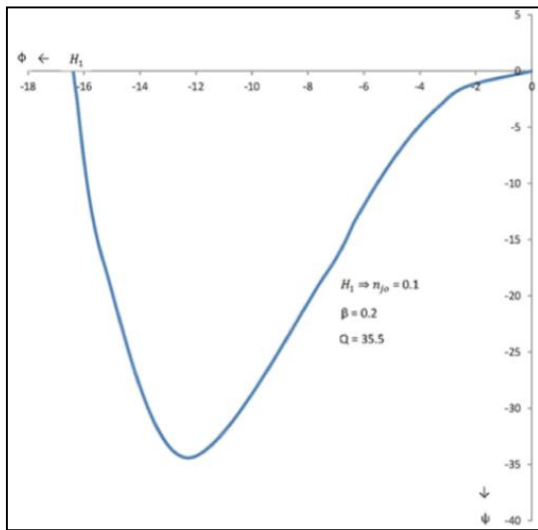
**Figure 3(a):** ψ verses ϕ for $n_{jo} = 0.1$, $\beta = 0.2$ **Figure 3(b):** ψ verses ϕ for $n_{jo} = 0.5$, $\beta = 0.2$

For (H^+, Cl^-) plasma with negative ion concentration $n_{jo} = 0.1$, rarefactive solitary waves are formed and the amplitudes of these rarefactive solitary waves are larger than the amplitudes of the rarefactive solitary waves for the same plasma at negative ion concentration $n_{jo} = 0.5$ in presence of non-thermal electron parameter(β). Moreover it is evident that when the negative ion concentration (n_{jo}) upto a certain limit is increasing, the amplitude is then decreasing. In this case the rarefactive solitary waves are represented by the figures H_1 (for $n_{jo} = 0.1$) and H_2 (for $n_{jo} = 0.5$) at $\beta = 0.2$ and $Q = 35.5$ which are shown by the Figs. 3(a) and 3(b). Again in absence of non-thermal electron parameter ($\beta = 0$), the sagdeev potential function $\psi(\phi)$ is changed with increasing concentration of negative ion[19] (from $n_{jo} = 0.1$ to 0.5) for $Q = 35.5$.
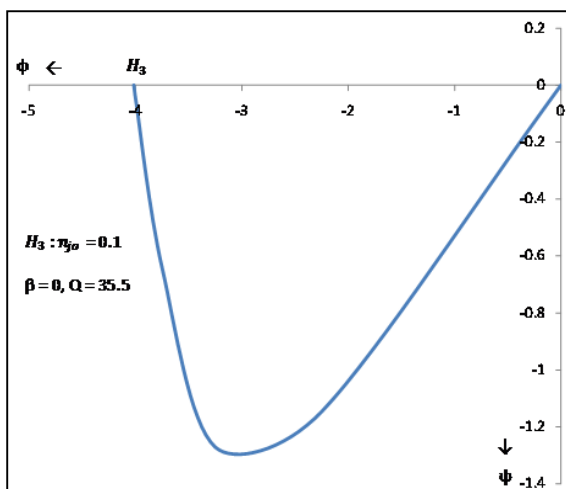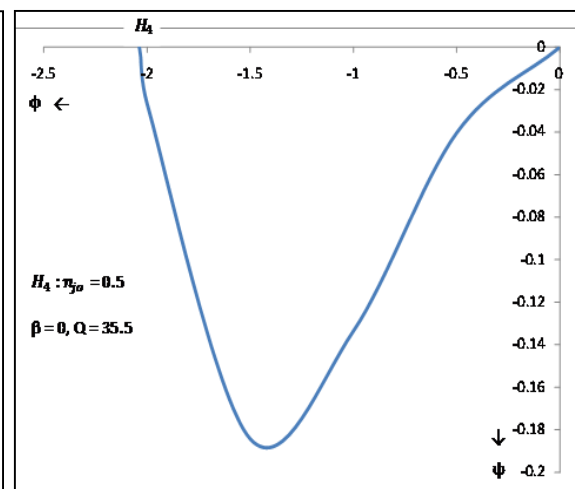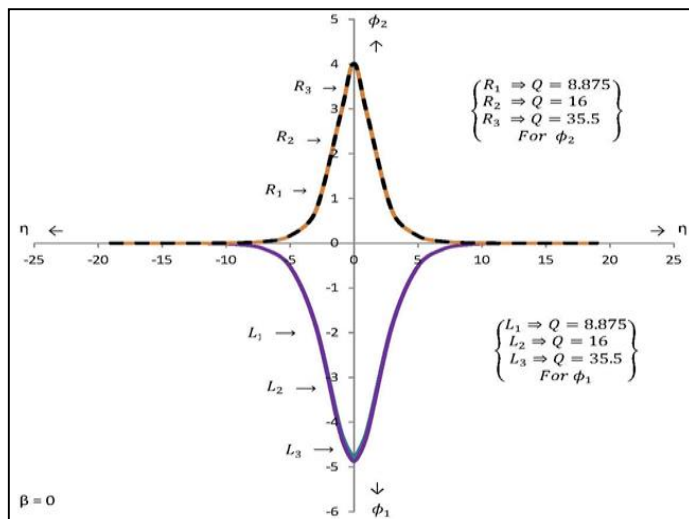

Figure 3(c): ψ verses ϕ for $n_{jo} = 0.1$, $\beta = 0$ **Figure 3(d):** ψ verses ϕ for $n_{jo} = 0.5$, $\beta = 0$

The rarefactive solitary waves in this case are represented by the figures H_3 (for $n_{jo} = 0.1$, $\beta = 0$ with $Q = 35.5$) and H_4 (for $n_{jo} = 0.5$, $\beta = 0$ with $Q = 35.5$) which are shown in the Figures 3(c) and 3(d). It is also observed from those figures that $H_2 < H_1$ and $H_4 < H_3$.

Figure.4 represents the structures of first (ϕ_1) and second (ϕ_2) order solitary wave solutions verses η(η is the Galelian transformation parameter) with the variation of the ratios of heavier negative to positive ion masses (Q) plasma. In absence ($\beta = 0$) and in presence ($\beta \neq 0$) of non-thermal electron parameter β, first (ϕ_1) order solitary wave solutions are negative whereas second order (ϕ_2) solitary wave solutions are positive for different increasing values of the mass ratios Q.

Figure 4(a): ϕ_1 and ϕ_2 verses η with Q variation for $\beta = 0$.

In a particular mass ratio (Q), the absolute values of first (ϕ_1) and second (ϕ_2) order solitary wave solutions are decreasing in presence ($\beta \neq 0$) and in absence ($\beta = 0$) of non-thermal electron. For Q = 8.875, 16 and 35.5, first (ϕ_1) order solitary wave solutions are denoted by the respective figures N_1, N_2 and N_3 in presence ($\beta \neq 0$) of non-thermal electron parameter β ($\beta = 0.2$) and those by L_1, L_2 and L_3 in absence of β($\beta = 0$). It is evident from the figures that $L_1 < L_2 < L_3$, $N_1 < N_2 < N_3$ and $N_1 < L_1$, $N_2 < L_2$ and $N_3 < L_3$ are valid upto a certain value of η and after that the result is changed.
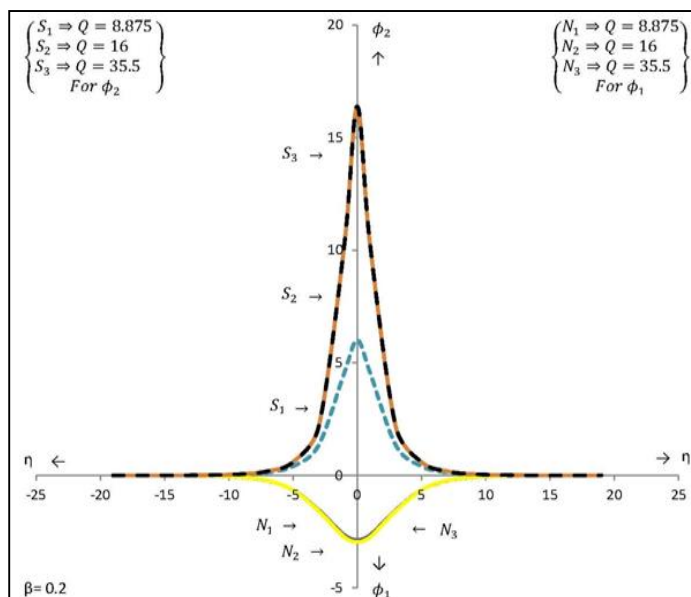
Figure 4(b): ϕ_1 and ϕ_2 verses η with Q variation for $\beta = 0.2$

Again for $Q = 8.875$, 16 and 35.5, the second (ϕ_2) order solitary wave solutions are denoted by S_1, S_2 and S_3 in presence of non-thermal electron parameter β ($\beta = 0.2$) and those by R_1, R_2 and R_3 in absence of $\beta(\beta = 0)$. It is also observed similarly from the figures that $R_1 < R_2$ $< R_3$, $S_1 < S_2 < S_3$ and $S_1 > R_1$, $S_2 > R_2$ and $S_3 > R_3$ in presence and in absence of β. These are represented by the figures 4(a) and 4(b). The interesting situation in this case is that all values of ϕ_2 with $\beta = 0.2$ for different η are larger than that of all second order values (ϕ_2) with $\beta = 0$ for different η in second order (ϕ_2) solitary wave solution whereas these results are just reverse like those of first order (ϕ_1) solitary wave solution.

3.   CONCLUSION

We have investigated theoretically the Sagdeev potential function (ψ), first (ϕ_1) as well as second (ϕ_2) order solitary wave solutions of heavier masses of negative ion plasma (Q). In this case we mainly studied the higher masses of negative ions with respect to positive ions. For higher mass ratio(Q), the effect of β on Q is an interesting new result. Rarefactive solitary waves are formed for this heavier mass ratio (Q) and the amplitude of this wave will be higher for larger mass ratio (Q) with higher negative ion concentration (n_{j0}). The first (ϕ_1) and second (ϕ_2) order rarefactive solitary wave solutions are larger for higher mass ratio (Q) in a particular non-thermal electron parameter(β).

Our future plan is to solve the same non-thermal plasma with relativistic positive and negative ion.

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